Nonlinear Elasticity Imaging: Theory and Phantom Study

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Abstract—In tissue the Young's modulus cannot be assumed constant over a wide deformation range. For example, direct mechanical measurements on human prostate show up to a threefold increase in Young's modulus over a 10% deformation. In conventional elasticity imaging, these effects produce strain-dependent elastic contrast. Ignoring these effects generally leads to suboptimal contrast (stiffer tissues at lower strain are contrasted against softer tissues at higher strain), but measuring the nonlinear behavior results in enhanced tissue differentiation.

To demonstrate the methods extracting nonlinear elastic properties, both simulations and measurements were performed on an agar-gelatin phantom. Multiple frames of phase-sensitive ultrasound data are acquired as the phantom is deformed by 12%. All interframe displacement data are brought back to the geometry of the first frame to form a three-dimensional (3-D) data set (depth, lateral, and preload dimensions). Data are fit to a 3-D second order polynomial model for each pixel that adjusts for deformation irregularities. For the phantom geometry and elastic properties considered in this paper, reconstructed frameto-frame strain images using this model result in improved contrast to noise ratios (CNR) at all preload levels, without any sacrifice in spatial resolution. From the same model, strain hardening at all preload levels can be extracted. This is an independent contrast mechanism. Its maximum CNR occurs at 5.13% preload, and it is a 54% improvement over the best case (preload 10.6%) CNR for frame-to-frame strain reconstruction. Actual phantom measurements confirm the essential features of the simulation.

Results show that modeling of the nonlinear elastic behavior has the potential to both increase detectability in elasticity imaging and provide a new independent mechanism for tissue differentiation.

I. INTRODUCTION

ARGER deformations may have significant advantage for elasticity imaging of soft tissue. For example, high internal strain over the entire region of an elasticity image can maximize the signal-to-noise ratio of the measured strain field [1]. However, at larger deformations, most tissues exhibit significant strain hardening, and the Young's modulus of tissue can no longer be considered constant [2]-[4]. Direct measurements on human prostate, using a

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Fig. 1. Two-layer phantom; force-deformation curve for materials (left) and phantom illustration (right). Data for the solid curve are based on direct mechanical measurements of cortex/medulla of canine kidney; the dashed curve is for the collecting system.

system similar to [5], showed that, even within a modest 10% deformation range, there is an approximately threefold increase in Young's modulus. Strain hardening can significantly reduce the strain contrast between different tissue types.

To illustrate the effect of strain hardening on image contrast, consider elasticity imaging based on a linear one-dimensional (1-D) model of a two-layer phantom. Fig. 1(left) shows the force-deformation relationship for the materials in this phantom, with the solid curve representing the top layer and the dashed curve the bottom layer. In Fig. 1(right) phantom deformation between two plates with a 1 N applied force is illustrated. The solid curve in Fig. 1(left) indicates 7% strain at this force for the stiffer top layer and the dashed curve 12.5% strain for the softer bottom layer, resulting in a 9.75% overall deformation.

Fig. 2 shows the effective Young's modulus (i.e., slope of differential stress versus differential strain at a particular strain value) for the layers in the phantom. The elasticity of the stiff top layer is measured at a relatively low 7% strain; the elasticity of the softer bottom layer is measured at 12.5% strain. In general, stiffer regions at low strain are contrasted against softer regions at higher strain, which in the presence of strain-hardening leads to reduced elastic contrast. The material properties used in Figs. 1 and 2 are based on direct mechanical measurements of collecting system and cortex/medulla samples of canine kidney [5].

In reality, the effects demonstrated here with a simple 1-D model take place in 3-D, and thus are more complicated. Full elasticity reconstruction relies on numerical solution of the partial differential equations describing the mechanical equilibrium of a deformed medium in which simplifications can be made assuming incompressibility and plain

Manuscript received February 20, 2003; accepted December 29, 2003. This work was funded by NIH grant DK47324.



Fig. 2. Two-layer phantom, Young's modulus. Data for the solid curve are based on direct mechanical measurements of cortex/medulla of canine kidney; the dashed curve is for the collecting system.

strain states. Clearly, strain hardening is an important tissue property and ignoring it may hurt image quality.

In addition to compensating for artifacts caused by strain hardening, the phenomenon itself can be measured and used to enhance tissue differentiation. In effect, it provides another contrast mechanism. In earlier experiments [6], artificial lesions were created in a canine kidney with gluteraldehyde. Mechanical measurements on these lesions revealed that the cross-linking agent not only increased the Young's modulus, but also reduced strain hardening. In [7], nonlinear elastic phantoms based on agar and gelatin were constructed that exhibited a complete contrast reversal, and a first-order fit procedure was performed to image the average strain-hardening behavior over a 12% deformation range. Varghese et al. [8] theoretically analyzed the effect of nonlinearity on CNR in a frame-to-frame strain image of a cylindrical inclusion. Contrast was calculated through a contrast transfer efficiency model [9], and noise characterized by a strain-filter model [10]. The optimal CNR frame-to-frame applied strain could be determined, and nonlinearity images were formed by subtracting frame-to frame strain images at low and high preload levels. Further refinements to the agar-gelatin phantoms were made, and a finite-element model of the phantom was developed [11] to verify our nonlinear elasticity algorithms. In-vivo experiments by Hall et al. [12] showed that fibroadenoma in the human breast gets relatively softer compared to surrounding tissue when preloads are increased, thus also exhibiting reduced strain hardening. Strain-hardening behavior alone can differentiate these lesions from surrounding tissue, and it can be a valuable adjunct to elasticity imaging. In this paper we build on these preliminary studies and demonstrate that strain hardening can be extracted from large deformation data acquired over many frames with a realtime ultrasound system.

II. METHODS AND MATERIALS

Direct mechanical measurements were performed on agar (1.9% by weight) and gelatin (10%) by deformation of

Fig. 3. Direct mechanical measurements on agar (solid curve) and gelatin (dashed curve).

a cylindrical sample (d = 14.3 mm, h = 12 mm) between two plates, with a setup similar to that in [5].

The results in Fig. 3 show the agar exhibiting significant strain hardening compared to gelatin. A five-parameter Mooney-Rivlin strain-energy function [13], [14] was used to characterize the nonlinear behavior of the agar, and a two-parameter model was used for the nearly linear gelatin over this deformation range. Special numerical procedures [15] were used in extracting these parameters to account for inaccurate measurements at low preload levels. For gelatin, the two- and five-parameter models produce nearly identical strain energy functions that are a good match to the direct mechanical measurement, indicating that a two-parameter model is sufficient to describe the elastic behavior. Agar has a more nonlinear behavior, and only the five-parameter model provided a good fit. The parameters are input to a finite-element simulation (ABAQUS, version 5.5, Pawtucket, RI) mimicking a phantom deformation experiment.

Fig. 4 shows the phantom geometry. It consists of a block of gelatin (10% by weight) with a triangular shaped bar of agar (1.9%) by weight) embedded within it. Pure agar and pure gelatin materials have limited bonding at the material interface, and agar is prone to crack propagation. With the geometry described here, larger deformations can be applied before the phantom will break, and elastic contrast is primarily transferred to axial strain contrast due to the 45 degree angle of the material interface (i.e., shear-strain contrast is minimized). Also, near the material interface, axial strain and Young's modulus are inversely proportional [16] at small deformation levels (linear elastic approximation, plain strain assumption). This phantom was deformed approximately 12% between plates at the top and bottom, using a hyper-elastic incompressible plain-strain model [17]. Due to the plain-strain model. the simulation assumes the phantom has infinite size in 3-D. In a physical experiment, the 3-D is finite, and the imaging plane is chosen in the middle to more closely approximate the plain-strain condition. A mesh of 300 (axial) by 150 (lateral) elements was used for the 10 cm \times 10 cm





Fig. 4. Phantom geometry for simulation and experiment. Dotted box is region of interest for imaging.

plane. Results are shown for a 3.2 cm \times 3.2 cm region of interest marked by the dotted line in Fig. 4, corresponding to a reasonable imaging region in a real experiment when a linear array is used. The mesh size is chosen to roughly match the lateral spatial resolution of the simulation to the lateral speckle spot size in the ultrasound phantom and has twice the spatial resolution in the axial dimension.

Equal force increments (100 N/m^2) are applied, and the resulting displacement data are used to create 44 frames of synthetic radio frequency (RF) data (360 beams \times 1920 samples). The point-spread function (PSF) is based on a 35-mm aperture, 5.1 MHz center frequency array with 50% fractional bandwidth. Data are sampled at 20 MHz to a 20mm depth. A frequency domain-based method convolves the scatterers with the PSF. Phase sensitive speckle tracking results in 43 frame-to-frame measured displacement fields. All frame-to-frame displacement fields are brought back to the geometry of the first frame, using the accumulated displacement from frame 1 to that frame. Thus, a specific image location is always associated with the same physical phantom part throughout all frames, allowing easier examination of local material behavior. The resulting data can be viewed as a 3-D dataset with lateral (x), axial (y), and preload (ε) dimension.

When the frame-to-frame displacement for a single image location is examined as a function of preload, the shape of the curve depends not only on the elastic behavior of the phantom but also on the rate at which the phantom is deformed for different preload levels. In the case of free-hand data acquisition, the nonuniformity in deformation speed most likely would be the dominating factor in determining curve shape. To overcome this problem, the displacement fields are converted to relative displacement ratio's (RDR), defined as:

$$RDR_{x,y} = \frac{\Delta v_{x,y}}{\Delta v_{\varepsilon}},$$
(1)



Fig. 5. A 3-D polynomial fit is performed on the RDR data for each pixel over a local region in the X and Y dimensions and all of the preload dimension. Each frame in the sequence is warped to the geometry of frame 1, using the accumulated displacement from frame 1 to that frame. This results in 18 fit coefficients for each pixel in the image.

where $v_{x,y}$ is the axial (or lateral) displacement at a pixel, and v_{ε} is the average axial displacement at the bottom of the image. This effectively normalizes the frame-to-frame displacements; values will be in the 0 to 1 range for all frames independent of interframe strain levels.

As illustrated in Fig. 5, for each pixel a surrounding area and all data in the preload dimension are taken to form a 3-D data subset. A 3-D, limited maximum variation order 4 (MVO = 4), polynomial fit is performed. Data at the center of the x-y region are given more weight in the minimum least square fit procedure, and weighting is reduced toward the edges using a Hanning weighting window in the axial and lateral dimensions. Along the preload dimension, all data are weighted equally. The Hanning weighting ensures smooth changes of the fit coefficients moving from pixel to pixel. In the spatial frequency domain, it has lower side lobes compared to a rectangular weighting window. For each pixel there are now 18 fit coefficients modeling the surrounding area as:

$$RDR(x, y, \varepsilon) = c_1 + c_2 x + c_3 y + c_4 x^2 + c_5 x y + c_6 y^2$$
$$+ c_7 \varepsilon + c_8 x \varepsilon + c_9 y \varepsilon + c_{10} x^2 \varepsilon + c_{11} x y \varepsilon$$
$$+ c_{12} y^2 \varepsilon + c_{13} \varepsilon^2 + c_{14} x \varepsilon^2 + c_{15} y \varepsilon^2$$
$$+ c_{16} x^2 \varepsilon^2 + c_{17} x y \varepsilon^2 + c_{18} y^2 \varepsilon^2, \qquad (2)$$

where x, y, and ε are the lateral, axial, and preload dimensions. The midpoints for the x and y grids in the fit are set to 0 for each new pixel, and the preload grid is based on accumulated axial displacement at the bottom of the image.

When the derivative of the RDR along the axial dimension is taken, we obtain the relative strain ratio (RSR). The RSR at a pixel is defined as:

$$\operatorname{RSR}_{x,y} = \frac{\Delta \varepsilon_{x,y}}{\Delta \varepsilon_{\varepsilon}} = \frac{\frac{\Delta v_{x,y}}{\partial y}}{\frac{\partial v_{\varepsilon}}{y_0}} = \frac{\partial \operatorname{RDR}_{x,y}}{\partial y} \cdot y_0, \quad (3)$$

where $\varepsilon_{x,y}$ is the normal axial strain (or normal lateral strain) at a pixel, $\varepsilon_{\varepsilon}$ is the average normal axial strain in the image, and y_0 is the maximum depth of the image (depth used for v_{ε}). Values below 1 indicate relatively

soft areas; values above 1 indicate stiffer areas. The axial derivative is taken analytically on the polynomial model, resulting in lower noise compared to taking the derivative numerically on displacement data. As we are interested only in the derivative at the axial and lateral midpoints, x and y are set to 0 and the derivative of (2) with respect to y leads to the following expression for the relative strain ratio:

$$RSR(x, y, \varepsilon) = (c_3 + c_9\varepsilon + c_{15}\varepsilon^2) \cdot y_0, \qquad (4)$$

where x, y, and ε are the lateral, axial, and preload dimensions, and y_0 is the same constant as in (2).

The relative hardening (RH) is obtained by taking the negative derivative of the RSR along the preload dimension. The RH is an indicator of the rate at which a region becomes stiffer or softer compared to the surrounding area as total deformation increases. The sign reversal is for easier interpretation of the data; positive values indicate relative stiffening and negative values indicate relative softening.

$$\mathrm{RH} = -\frac{\partial \mathrm{RSR}}{\partial \varepsilon}.$$
 (5)

This derivative is again taken analytically on the polynomial of (4), leading to:

$$\operatorname{RH}(x, y, \varepsilon) = (c_9 + 2c_{15}\varepsilon) \cdot (-y_0).$$
(6)

Thus, to reconstruct RSR and RH images, only 3 of 18 coefficients are needed. The polynomial fitting procedure is based on minimizing the square error between data and fit by finding the coefficients for which the derivative of the error function is 0. This leads to 18 equations with 18 unknowns, and generally requires the inversion of an 18×18 matrix. However, the perfect antisymmetry in the x and y grids cause exact cancellations for many terms in the set of equations (a sparse 18×18 matrix). The equations decouple into four independent groups involving $(c_2, c_8, c_{14}), (c_3, c_9, c_{15}), (c_5, c_{11}, c_{17}),$ $(c_1, c_4, c_6, c_7, c_{10}, c_{12}, c_{13}, c_{16}, c_{18})$. Only the second set of coefficients is needed, and for pixel location X, Y can be obtained using the matrix equation (7) (see next page). The summations are over the local 3-D space surrounding a pixel as shown in Fig. 5, in which the pixel is always made the midpoint (x = 0, y = 0). In addition, the summations are Hanning weighted in x and y dimensions [weighting is not shown in (7)]. Any weighting functions on the summation are allowed, as long as they are symmetric in x and yaround the middle point (x, y = 0, 0).

Note that all variables in the matrix are independent of the pixel chosen (pixel is always made midpoint of xand y grid), so the inverse matrix needs to be calculated only once. The vector on the right does depend on pixel position (variables X, Y) due to changing RDR functions. The $y, y\varepsilon$, and $y\varepsilon^2$ terms are precalculated once, so each pixel needs three multiply accumulate operations over the small region shown in Fig. 5. This is a fairly low computational cost, negligible compared to the speckle tracking computations.



Fig. 6. Relative strain ratio (RSR) and relative hardening (RH) images based on simulated displacements in an agar and gel phantom. Rows are for preloads of 0.2%, 5.13%, and 10.6%, respectively. The left column has raw frame-to-frame strain images normalized to RSR. The center column is the RSR reconstructed from 3-D polynomial fit coefficients, and the right column is the RH image reconstructed from fit coefficients. The RSR images are displayed over a 0.5–1.4 dynamic range, and the RH images are displayed over a -15 to 15 dynamic range. Dotted boxes in the bottom right image indicate regions used for CNR calculations.

A real phantom experiment was performed on a Siemens elegra using a 5.1 MHz linear array, collecting 36 RF frames (360 beams × 920 samples) in a 4 × 4 cm imaging region. The phantom has extra fine graphite powder (1.3% by weight, average particle size of 9 μ m) as an ultrasonic scattering material, and bleach (0.2% by weight) is added as a preservative. Gelatin (10% by weight) is poured first, and after setting serves as a mold for the agar. By cooling the gelatin to 5°C and pouring the agar (1.9% by weight) at 60°C, slight mixing of phantom materials near the boundary occurs. In addition, the gelatin has 1-mm grooves at its boundary to further facilitate mixing; this is necessary for sufficient bonding between phantom materials.

III. Results

Fig. 6 shows images based directly on simulated displacement data from ABAQUS over a 3.2-cm \times 3.2-cm region of interest (Fig. 4). The left column has frame-to-frame strain images scaled to conform to RSR images at preloads of 0.2%, 5.13%, and 10.6%. All displacement data were brought back to the geometry of the first frame. Strains were calculated using a local 2-D, first-order polynomial fit around each pixel, giving a 1.6-mm \times 1.4-mm

$$\begin{bmatrix} c_{3} \\ c_{9} \\ c_{15} \end{bmatrix} = \begin{bmatrix} \sum_{x=-\Delta x}^{\Delta x} \sum_{y=-\Delta y}^{\Delta y} \sum_{\varepsilon=0}^{\varepsilon_{\max}} y^{2} & \sum_{x=-\Delta x}^{\Delta x} \sum_{y=-\Delta y}^{\Delta y} \sum_{\varepsilon=0}^{\varepsilon_{\max}} y^{2}\varepsilon & \sum_{x=-\Delta x}^{\Delta x} \sum_{y=-\Delta y}^{\Delta y} \sum_{\varepsilon=0}^{\varepsilon_{\max}} y^{2}\varepsilon^{2} \\ \sum_{x=-\Delta x} \sum_{y=-\Delta y}^{\Delta x} \sum_{z=0}^{\Delta y} \sum_{x=0}^{\varepsilon_{\max}} y^{2}\varepsilon^{2} & \sum_{x=-\Delta x}^{\Delta x} \sum_{y=-\Delta y}^{\Delta y} \sum_{\varepsilon=0}^{\varepsilon_{\max}} y^{2}\varepsilon^{3} \\ \sum_{x=-\Delta x} \sum_{y=-\Delta y}^{\Delta x} \sum_{z=0}^{\Delta y} \sum_{x=0}^{\varepsilon_{\max}} y^{2}\varepsilon^{2} & \sum_{x=-\Delta x}^{\Delta x} \sum_{y=-\Delta y}^{\Delta y} \sum_{\varepsilon=0}^{\varepsilon_{\max}} y^{2}\varepsilon^{4} \end{bmatrix} \\ \cdot \begin{bmatrix} \sum_{x=-\Delta x} \sum_{y=-\Delta y}^{\Delta x} \sum_{\varepsilon=0}^{\Delta y} \sum_{z=0}^{\varepsilon_{\max}} y^{2}\varepsilon^{2} \\ \sum_{x=-\Delta x} \sum_{y=-\Delta y}^{\Delta x} \sum_{\varepsilon=0}^{\Delta y} \sum_{z=0}^{\varepsilon_{\max}} x \sum_{z=0}^{\Delta y} \sum_{z=0$$

axial by lateral spatial resolution. At the low 0.2% preload level, the agar is much softer than the gelatin, resulting in a high, relative-strain ratio for the inclusion. Increasing the preload to 5.13% and 10.6% causes strain hardening in the agar, and complete contrast reversal takes place. Small irregularities can be seen in the RSR images. They are caused by the limited numerical precision of the non-linear, finite-element simulation and have a nearly random behavior.

The center column in Fig. 6 shows RSR images, based on the analytical derivative of a 3-D polynomial fit to the RDR data. The axial and lateral window sizes used for the 3-D fit were chosen such that the spatial resolution of the left and center columns would be identical. Note that these images are nearly identical to the left column, except that the numerical precision irregularities are absent. Analyzing the difference between the left and center columns showed randomly behaving zero mean noise with increased magnitude at depth; no bias was introduced.

The right column in Fig. 6 shows the RH images; it is the analytic derivative of the RSR sequence in the center column with sign reversal (6). The spatial resolution matches that of the first two columns. The agar is initially soft with high RSR values that decrease rapidly under higher preload. Thus the slope of the agar RSR starts out large negative (~ -15) leading to positive (~ 15) RH values. The gelatin RSR starts out low and increases, giving positive slopes (negative RH). For this particular phantom geometry and material behavior, the highest RH contrast is achieved at low preload, and contrast is gradually reduced as preload is increased.

The highest contrast for RSR is at the lowest and highest preload levels, and the best contrast for the RH image is at low preload. However, what really matters is the CNR. In the presence of tracking noise, optimal preload levels may change. Thus, synthetic RF data were generated and speckle tracking was performed to find the displacement fields. As the real displacements for the synthetic RF data are known, accurate CNR analysis could be performed. Also, as the exact error signal is known, the width of its

TABLE I

CONTRAST-TO-NOISE RATIOS FOR SYNTHETIC RF DATASET.¹

Preload [%]	0.2	1.37	2.82	5.13	7.25	10.6
CN raw RSR	1.47	0.98	0.90	0.88	1.30	2.22
CNR fit RSR	4.21	2.59	0.45	2.08	3.75	5.21
CNR fit RH	4.89	5.47	6.49	8.03	5.36	1.61

¹Rows have raw frame-to-frame strain, relative strain ratio reconstructed from fit, and relative hardening reconstructed from fit. Columns are for increasing levels of preload. Corresponding images are in Fig. 7.

autocorrelation at half-maximum can be used to find the spatial resolution.

The dashed lines in the bottom right image of Fig. 6 show the two regions used in the CNR calculations. The formula used for CNR calculation is:

$$CNR = \frac{|m1 - m2|}{\sqrt{\mathrm{std1}^2 + \mathrm{std2}^2}},\tag{8}$$

where m1 and m2 are mean signal levels in the areas contrasted, and std1 and std2 are the standard deviations of the noise in those areas.

Fig. 7 shows the RSR and RH images based on synthetically generated RF data. All images are over the same region of interest and at the same spatial resolution (1.6 mm \times 1.4 mm) used in Fig. 6. Comparing the raw frame-to-frame strains in the left column to the fit-based RSR images in the center column, noise reduction at all preload levels was achieved. The right column with RH images has its highest contrast at low preload. However, as preload was increased, the noise decreased at a faster rate than the contrast, and the optimal CNR was achieved at 5.13% preload.

Table I shows the CNR values for the images in Fig. 7 (see Fig. 6 for regions used in CNR calculation). For the RSR images, the maximum CNR levels are at 0.2% and 10.6% preload, and the CNR is improved by more than a factor of 2 over their raw frame-to-frame strain counterparts. The best case RH image is at preload 5.13%. Its CNR of 8.03 is a nearly fourfold improvement over the



Fig. 7. Relative strain ratio (RSR) and relative hardening (RH) images based on synthetically generated RF data from an agar and gel phantom. Rows are for preloads of 0.2% to 10.6% as noted. The left column has raw frame-to-frame strain images normalized to produce the RSR. The center column is the RSR reconstructed from 3-D polynomial fit coefficients, and the right column is the RH image reconstructed from fit coefficients. The RSR images are displayed over a 0.5 to 1.4 dynamic range, and the RH images are displayed over a -15 to 15 dynamic range.

RSR image at that preload (nine times better than raw strain). At all but the highest preload level, RH outperforms RSR. Best case RH beats best case RSR by a factor 1.5 and best case raw strain by a factor of 3.6. These results are, of course, for the specific geometry and phantom materials considered in this paper. In general, RH is expected to outperform RSR when some material has significantly more strain hardening than surrounding areas; but, for low preload levels it is not significantly stiffer than the surrounding area. As RH provided superior contrast, the trade-off between CNR and spatial resolution was investigated.

Fig. 8(a) shows the best-case RH at a spatial resolution of $0.6 \text{ mm} \times 0.6 \text{ mm}$. This is approximately the spatial resolution of the RF data (the speckle spot size), and matches the Hanning weighted kernel and filter sizes used in the



Fig. 8. Relative hardening image from synthetically generated RF data, at preload of 5.13%: (a) high resolution (0.6 mm \times 0.6 mm); (b) low resolution (1.6 mm \times 1.4 mm).

correlation processing. Even at a resolution matching the imaging system, a CNR of 2.92 is obtained, outperforming the frame-to-frame strains at much lower resolution shown in Fig. 7. Fig. 8(b) shows the best case RH at a spatial resolution of 1.6 mm \times 1.4 mm (axial, lateral), with a CNR of 8.03. Clearly, RH imaging can be useful in both CNR and spatial resolution improvement.

A real agar/gelatin phantom was constructed to further test the algorithms. Compared to the simulated geometry, the inclusion is a little closer to the transducer, and the angle at the tip of the inclusion is slightly larger than 90 degrees. These deviations are caused in part by melting and remixing of the gelatin when the agar inclusion is poured. It should be noted that, at the material boundaries, there is a significant area containing both agar and gelatin. Furthermore, the Young's modulus of gelatin has a strong temperature dependence around room temperature, and the real phantom is not a pure plane strain case (although close). Sound speeds in the phantom materials are not equal [8], causing slight diffraction of the imaging beam. Due to these factors, an exact match with the simulation should not be expected; the experiment merely is a qualitative comparison to theory.

Fig. 9 shows best case results; Figs. 9(a) and (b) are RSR at low and high preload, and Fig. 9(c) is RH at 6.36% preload. At the low and high preloads, high-stress regions are generated near the boundary due to strong differences in material behavior, and imperfect bonding may be responsible for the high RSR values seen there. Because the real (not measured) displacements needed to calculate exact CNR numbers are not available, 2-D, second-order polynomial fits in the regions used for CNR calculation are used as references instead. As a test, that same CNR calculation method also was applied to synthetic data, and found to always be within 5% of the exact CNR values. For low (0.6%) and high (11.9%) preload, the CNR of the RSR are 1.81 and 1.97, respectively: the RH image has a CNR of 2.48. The CNR for raw frame-to-frame strain images at preloads of 0.6%, 6.36%, and 11.9% are 1.71, 0.35, and 1.47, respectively. The best case RH has a 26% improvement over the best case RSR and 45% improvement over the best-case, raw frame-to-frame strain.



Fig. 9. Experiment on real agar and gelatin based phantom: (a) relative strain ratio at 0.6% preload; (b) relative strain ratio at 11.9% preload; (c) relative hardening at 6.4% preload.

IV. DISCUSSION

This work demonstrates that modeling of the nonlinear elastic behavior can increase differentiation between different material types. Larger deformations can increase SNR [1], [18], but due to nonlinear elastic effects decrease contrast and CNR as well. Nonlinear processing can overcome this limitation, improving the quality of traditional frame-to-frame strain images. In addition, a new contrast mechanism of relative hardening is a natural product of this processing.

All results presented here were for a two-component phantom consisting of a nearly linear material (gelatin) and a nonlinear material (agar) [19]. For this simple system, the simulations show that the best case CNR for frame-to-frame strain images is improved more than twofold (2.22 to 5.21), and imaging RH can generate a CNR of 8.03. This is all without sacrificing spatial resolution. Alternatively, nonlinear modeling can be used to increase spatial resolution. Increasing the resolution to the size of the B-scan speckle spot still leads to a better CNR for RH compared to best case raw strain while reducing the area of the resolution cell by a factor of 6.

This paper only shows RSR and RH images at several preload levels; for best results, movies of RSR and RH (where preload is encoded as time) should be used. The preload level at which optimal CNR is achieved depends both on material behavior and geometry, and thus can change for different regions of the image. Movies capture all this information.

Future work will include direct mechanical measurements on tissue to identify pathologies in which nonlinear elasticity imaging can be particularly effective in enhancing CNR. Further improvements in the algorithm can be made through adaptive weighting schemes in the 3-D fit. Frame decimation techniques can be used to optimize the frame-to-frame strain for each region in the image and improve the SNR [20]. The ultimate goal of elasticity imaging is to obtain scalar parameters describing the nonlinear elastic behavior for each pixel in the image, from local elasticity reconstructions independent of boundary conditions.

Acknowledgment

The authors thank Siemens Corporation for support through equipment and cooperation.

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